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# Derivation of dynamical equations of Cosmology and some conclusions

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**Abstract :**

*In this paper, after derivation of the dynamical equations of cosmology, some conclusions will be made.*

**Introduction :**

The field equations given by Einstein when combined with the isotropic, homogeneous line elements give us the dynamical equations of cosmology.

These equations are satisfied by the scale function $R(t)$. The corresponding components of the metric tensor for the line elements

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right\}$$

are

$$\left.\begin{aligned} g_{00} &= 1 = g^{00}, g_{11} = -\frac{R^2}{1 - kr^2} = (g^{11})^{-1} ; \\ g_{22} &= -R^2 r^2 = (g^{22})^{-1} ; \\ g_{33} &= \sin^2\theta, g_{22} = (g^{33})^{-1} \end{aligned}\right\} \qquad (1)$$

From the relation above and from the relation

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left( \frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} + \frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right),$$

We have

$$\left.\begin{aligned} &\Gamma^{0}_{ij} = -\frac{1}{2}\frac{\partial}{\partial t} g_{ij}\, ; \Gamma^{i}_{0j} = \frac{\dot{R}}{R}\delta^{i}_{j} \\ &\Gamma^{1}_{11} = \frac{kr}{1-kr^{2}}\, ; \Gamma^{1}_{22} = -r(1-kr^{2})\, ; \\ &\Gamma^{1}_{33} = \sin^{2}\theta\, \Gamma^{1}_{22} \\ &\Gamma^{2}_{12} = \Gamma^{3}_{13} = \frac{1}{r}\, ; \Gamma^{2}_{33} = -\frac{1}{2}\sin^{2}\theta \\ &\Gamma^{3}_{33} = \cot\theta \end{aligned}\right] \qquad (2)$$

All other components of $\Gamma$ either vanish or follow from the symmetry property

$$\Gamma^{\lambda}_{\nu\mu} = \Gamma^{\lambda}_{\mu\nu}$$

From (2), we calculate the Ricci tensor by using

$$R_{\mu\nu} = \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{\nu}}, \Gamma^{\sigma}_{\mu\nu}\Gamma^{\lambda}_{\sigma\lambda} - \Gamma^{\sigma}_{\mu\lambda}\Gamma^{\lambda}_{\sigma\nu}$$

The non-vanishing components of which are easily found to be

$$R^{0}_{0} = -3\frac{R}{R}\, ; R^{i}_{j} = -\frac{1}{R^{2}}(R\ddot{R} + 2\dot{R}^{2} + 2k)\delta^{i}_{j} \qquad (3)$$

Now let us assume that the cosmological fluid is an ideal fluid. According to

$$T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho)U_{\mu}U_{\nu}; g_{\mu\nu}U^{\mu}U^{\nu} = 1$$

the energy momentum tensor, therefore, is

$$T^{\nu}_{\mu} = - p\delta^{\nu}_{\mu} + (p + \rho)U^{\nu}U_{\mu};\ U_{\nu}U^{\nu} = 1 \tag{4}$$

where $p$ is the pressure and $\rho$ the energy density of the cosmological fluid.

The fluid is at rest in the co-moving frame, accordingly

$$\left.\begin{aligned} u^{i} &= 0 = u_{i} \\ u^{0} &= 1 = u_{0} \end{aligned}\right\} \tag{5}$$

Now, we can apply the Einstein equations :

$$R^{\nu}_{\mu} = - \frac{1}{2} \int^{\nu}_{\mu} R^{\lambda}_{\lambda} = 8\pi\, T^{\nu}_{\mu}$$

The time - time component (i.e. $\mu = 0$, $\nu = 0$) of the above gives us

$$\dot{R}^{2} + k = \frac{8\pi}{3}\rho R^{2} \tag{6}$$

while the space- space components give us

$$2R\ddot{R} + \dot{R}^{2} + k = -8\pi\, pR^{2} \tag{7}$$

The above equation inform us that the pressure and density are independent of spatial co-ordinates.

The Bianchi identity requires that

$$T^{\nu}_{\mu;\nu} = 0$$

Writing this out in full, we get :

$$\frac{\partial p}{\partial x^{\lambda}} - \frac{\partial}{\partial x^{\nu}} \Big\{ (p + \rho)U^{\nu}U_{\lambda} - (p + \rho)\, \Gamma^{\nu}_{\sigma\nu} U^{\sigma}U_{\lambda} + (p + \rho)\, \Gamma^{\sigma}_{\nu\lambda} U^{\nu}U_{\sigma} \Big\} = 0 \tag{8}$$

Simplifying the above using (2) and (5), we have

$$\frac{\partial p}{\partial x^{\lambda}} - \frac{d}{dt}\{(p+\rho)U_{\lambda}\} - 3(p+\rho)\frac{\dot{R}}{R}U_{\lambda} = 0 \tag{9}$$

The non-trivial part of the above equation corresponding to $\lambda = 0$, is

$$\dot{\rho} + 3(p+\rho)\frac{\dot{R}}{R} = 0 \tag{10}$$

The equations (6), (7) and (10) and all independent. If we take the derivatives of (6), we easily get the equation (7), by using (10).

We shall take (6) and (10) as our independent equations.

Thus the equations of cosmology are

$$\dot{R}^2 + k = \frac{8\pi}{3}\rho R^2$$

and $\quad 2R\ddot{R} + \dot{R}^2 + k = -8\pi p R^2$

**Conclusions :**

We have

$$\dot{R}^2 + k = \frac{8\pi}{3}\rho R^2 \tag{I}$$

$$\dot{\rho} + 3(p+\rho)\frac{\dot{R}}{R} = 0 \tag{II}$$

Taking derivative of the first equation and eliminating $\dot{\rho}$ with the aid of the second, we have

$$\ddot{R} = -\frac{4\pi}{3}(\rho + 3p)R \tag{III}$$

The above means that the expansion of the universe is getting deaccelerated due to the presence of matter in the universe.

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